Problem 2

4a)

We will construct a set of size d in d dimension, such that it can be shattered.

For a point in the set , assign the bit value 1 and everything else value 0. For example for , . Then we’ll show that if we want to classify k points as positive: . Then a sphere of radius 1 centered at , where , will contain all points in and exclude all points in .

There are two things to prove with the construction:

1. All points in is not contained in the sphere

Notice that by construction, all points are orthogonal to each other. Therefore a linear combination of points in (call this ) will be orthogonal to any points in . Then note that the distance between and a point in will be since . So, the sphere will no longer contain a point in .

1. All points in is contained in the sphere

Again, we use the fact that all points are orthogonal. Call a linear combination of points in (same as before). The distance from the center of the sphere to a point in :

Since we constructed , then

So we know that .

Therefore,

which means a sphere of radius 1, centered at will contain it.